Tarski and the Semantic theory of Truth

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22.10.2022

Truth and Meaning

In the history of philosophy, a number of different theories have been proposed about truth. We shall briefly gloss over them:

- **The Coherence Theory of truth:** It states that a proposition is true if and only if that proposition coheres with the other propositions that one believes. By the standards of the coherence theory, a belief is true if and only if that belief is consistent with one's other beliefs. Truth is then a matter of the logical relations between believed propositions.

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- **The Correspondence Theory of Truth:** It states that a belief is true if and only if that belief corresponds to the facts. It is called such as it talks about the relationship between a proposition and something in the world outside of the proposition—facts or states of affairs. There are these things out there in the world and a true proposition is one that corresponds to them.

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- **Pragmatic Theory of truth**: It states that a proposition is true if and only it is useful to believe in that proposition. That is, a proposition is true if and only if one's plans and projects go better by believing it than by not believing it. Truth is utility.

Objections

We shall now see some objections to these proposed theories of truth.

- **Coherence theory:** The problem with this theory is that a belief could be consistent with other beliefs and yet the whole lot could be false. Consistency alone is not sufficient to make a belief true, As false propositions can be mutually consistent. For example, the belief that the Earth is flat is consistent with the belief that you will drop off the edge if you travel far enough, although neither belief is true.

Objections

- **Pragmatic theory:** This theory has similar objections in the sense that one could have beliefs which are useful while they may not be true. For example, in Soviet Russia, the belief that the bourgeoisie are wicked, you are likely to be rewarded (or otherwise, executed). It is beneficial to hold such a belief, but the truth condition of the statement does not follow from it.

Objections

- **Correspondence Theory:** While considered as the best theory which captures the idea of truth by many philosophers, the problem with this theory is more of a technical nature such as what a fact is and what the correspondence relation amounts to. Are facts complexes of objects and properties? How do we count them? How exactly do they differ from true propositions? It is also difficult to find a clear and correct formulation of the underlying notion of correspondence to reality.

Objections

The clarification of suh a correspondence theory, on a firm logical basis is what Alfred Tarski (arguably the best logician of his time along with his contemporary Gödel) sought out to do in his program, The Semantic theory of Truth (STT).

Truth and definability

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Definition (Definiability)

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Thus, definability is mathematically tractable only if truth is. But is truth tractable? The liar paradox, in which a contradiction is derived from seemingly undeniable premises about truth, may seem to suggest that it isn't.

Liar Paradox

P1. 'Sentence 1 is not true' is true iff Sentence 1 is not true.
P2. Sentence 1 = 'Sentence 1 is not true'
C. Sentence 1 is true iff Sentence 1 is not true

The paradox is generated by

1. The existence of a self-referential sentence that says of itself that it isn't true
2. The correctness of 'S' is true iff S, which seems to be guaranteed by the very meaning of 'true'.

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Since T is part of M but not L, no sentence containing it is one to which it applies, and no liar-sentence is constructible in either M or L

Construction of definition of truth

I shall illustrate a definition of truth for the language of arithmetic over First Order Logic (LA)

- Quantifiers of which range over $\mathbb{N}$
- Nonlogical vocabulary of which consists of the predicate '$=$', the name '0', the unary relation symbol 'S' (standing for successor), and the binary relation symbols '$+$' and 'x' (standing for addition and multiplication)

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$Q(\tilde{n})$ is a sentence that arises from the quantified sentence by erasing the quantifier and replacing all free occurrences of x in Q with occurrences of n.

Definition of truth in LA

Let us consider the following :

Tarski's Schema T: S is true$_{LA}$iff P*

Tarski required that for each sentence S of LA, an instance of **schema T** can be derived from the definition of $truth_{LA}$. Instances arise by replacing '$S*$' with a name of S, and replacing 'P' by a sentence of M that paraphrases S.

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Since we are justified in accepting every instance of *'If S* means that P, then S* is true iff P'*, we know that for every sentence S of LA, S is $true_L A$ iff S is true. Hence, Tarski's defined predicate is coextensive, over LA, with our ordinary truth predicate.

The preceding definition of truth is recursive. First, truth is defined for the simplest sentences. Then, the truth of complex sentences is defined in terms of the truth of simpler ones. This works because for each object (number) that LA is used to talk about, there is a variable-free term in LA that assigns it.

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The definition of truth which we constructed is rich enough to obtain results such as the following.

Theorem (Tarski's Undefinability Theorem)

There is no formula $T(n)$ of LA such that $T(n) \leftrightarrow A$ where n is the assigned number of A such that A is $true_{LA}$

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Final Remarks

Tarski's theory of truth uses logical tools throughout. But it is not a logical calculus in the sense in which propositional or predicate logic are. It is meta-mathematical, and eventually axiomatic, if the above approach is chosen.

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The philosophical implications of STT plays an important role in philosophy of language, logic and mathematics, in clarification of some issues.

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References

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Thank You